THE FLOW OF HEAT FROM THE CHIP-FORMATION ZONE TO THE WORKPIECE

Yu. P. Rasputin

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The problem of heat flow from the strain zone of metal to the workpiece in free orthogonal cutting is solved by the method of high-speed sources.

The cutting of metals is accompanied by heat release in the zone of plastic deformation (Fig. 1). Numerous publications [1, 3, 4, 6] have been devoted to the investigation of the heat flux from this zone to the workpiece. The authors of these investigations neglect either one of the components of the medium velocity, or one component of the latter together with the inclination of the source to the shear plane. A solution of this problem which takes more fully into consideration the kinematics of this process is presented here.

Let us consider a steady free orthogonal cutting process with plane deformation of metal and total heat, equivalent to the power spent on chip formation, released in a narrow zone. We shall consider the latter as a plane source of uniform intensity.

The heat flux reaching the workpiece from the strain zone can be calculated when the temperature field in the neighborhood of the shear line L−L is known. Let us determine this field.

It was shown in [2] that the process of plane heat propagation from a powerful high-speed linear source may be considered as the sum of independent one-dimensional heat propagation processes in infinite elementary rods. We shall make use of this property of the temperature fields of fast moving sources.

We separate in the workpiece a rectangular element (Fig. 1) and shall consider it as an infinite rod with heat-insulated sides. This rod moves through the source at velocity v_1 and at the same time travels along it at velocity v_2. The latter in conjunction with the geometrical parameters of the cutting process determines the time of heating the rod

$$t=\left(\frac{h}{\sin\beta}-y\right)v_2^{-1}. \tag{1}$$

The temperature field U(x, t) of the rod during its motion reproduces approximately the contour of the actual field in front of the strain zone.

The process of heat propagation in such a rod is defined by the Fourier−Kirchoff equation

$$c\gamma\left(\frac{\partial U}{\partial t}-v_1\frac{\partial U}{\partial x}\right)=\operatorname{div}(\lambda\operatorname{grad}U). \tag{2}$$

This equation is written in a system of coordinates attached to the source.

Taking this into consideration, we reduce the problem of the temperature field in the neighborhood of the shear line to the following: find a solution of the heat conduction equation (2) satisfying boundary conditions

$$U(x,\ 0)=\begin{cases}U_0 & \text{for}\quad x=0,\\ 0 & \text{for}\quad 0<x<+\infty,\end{cases} \tag{3}$$
$$U(0,\ t)=U_0.$$

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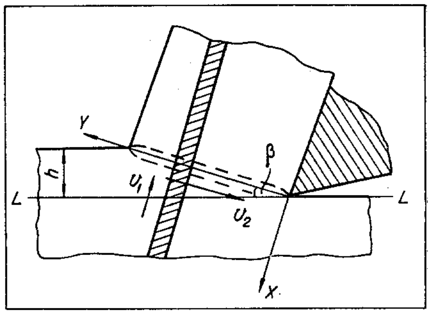

Fig. 1

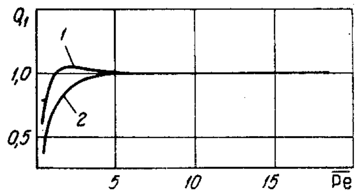

Fig. 2

Fig. 1. Diagram of the temperature field and heat flow (the strain zone is shown cross-hatched).

Fig. 2. Dependence of the dimensionless parameter Q_1 on the Peclet number [1) Q_1^*; 2) Q_1^{**}).

Known solutions of this problem do not take fully into consideration the kinematics of motion of the medium. For example, in [3] and [4] components v_2 and v_1 of the rod velocity are, respectively, assumed to be zero. Such solutions are particular cases of the stated problem.

The solution derived here takes into consideration both of these velocity components.

By the substitution

$$U(x,\ t) = T(x,\ t)\exp\left[-\frac{v_1}{2a}x - \frac{v_1^2}{4a}t\right] \tag{4}$$

Eq. (2) is reduced to the canonical form of the heat conduction equation

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}. \tag{5}$$

The boundary conditions for function T become

$$T(x,\ 0) = \begin{cases} U_0 & \text{for}\quad x = 0, \\ 0 & \text{for}\quad 0 < x < +\infty, \end{cases}$$
$$T(0,\ t) = U_0\exp\left[\frac{v_1^2}{4a}t\right]. \tag{6}$$

Function

$$T(x,\ t) = \frac{U_0 x}{2\sqrt{\pi a}}\int_0^t \frac{\exp\left(\dfrac{v_1^2\tau}{4a}\right)}{(t-\tau)^{3/2}}\exp\left[-\frac{x^2}{4a(t-\tau)}\right]d\tau \tag{7}$$

satisfies Eq. (5) and boundary conditions (6) [5].

Substituting (7) into (4), we obtain the looked-for solution

$$U(x,\ t) = U_0\frac{2}{\sqrt{\pi}}\exp\left[-\frac{v_1 x}{2a}\right]\int_{x/2\sqrt{at}}^{\infty}\exp\left[-\frac{\delta}{Z^2} - Z^2\right]dZ; \tag{8}$$

here

$$\delta = \left(\frac{v_1 x}{4a}\right)^2,\quad Z = \frac{x}{2\sqrt{a(t-\tau)}}.$$

Using the additivity property of the definite integral, we rewrite formula (8) in the form

$$U(x,\ t) = U_0\left[\exp\left(-\frac{v_1 x}{a}\right) - \exp\left(-\frac{v_1 x}{2a}\right)\frac{2}{\sqrt{\pi}}\int_0^{x/2\sqrt{at}}\exp\left(-\frac{\delta}{Z^2} - Z^2\right)dZ\right]. \tag{9}$$

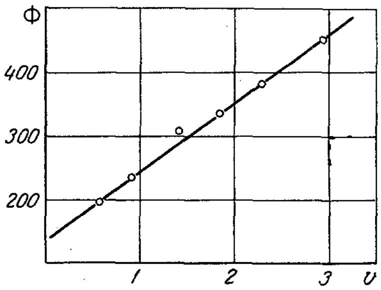

Fig. 3. Dependece of the heat flux Φ (W) to the workpiece on the cutting speed v (m/sec).

Since it is independent of time t, the first term of solution (9) defines in a system of coordinates attached to the source the stabilized temperature of the rod. The second term depends on time and represents the nonsteady temperature field superposed on the first. The v_1 velocity component appears explicitly in (9), and component v_2 in terms of the rod heating time. Temperature fields with one of the components of the medium velocity vanishing are particular cases of (9).

It is possible to calculate from a known temperature field the heat flux to the workpiece from the chip-formation zone. In the positive direction of the OX-axis it is

$$\tilde{\Phi}_1 = -\lambda b v_2 \int_0^t \left.\frac{\partial U}{\partial x}\right|_{x=0} dt.$$

Such flux would reach the workpiece when the angle β of inclination of the source is zero. In an actual cutting process β is not equal to zero. Hence the workpiece gets only the part

$$\Phi_1 = -\lambda b \int_L \frac{\partial U}{\partial x} \cos\beta \, dl \tag{10}$$

of flux $\tilde{\Phi}_1$ passing through the shear plane L. The remaining heat is carried away by the chip. The inclination of the source to the shear plane is taken into account in formula (10).

The derivative $\partial U/\partial x$ appearing in (10) is taken at the point of the rod moving along the shear plane. It can be found by differentiating (9). However formula (9) contains an integral which cannot be expressed in terms of elementary or tabulated special functions, which makes the calculation and analysis of heat flow difficult. It is, therefore, expedient to determine the nature and extent of the heat flux variation by approximate methods. For this we shall find two functions Φ_1^* and Φ_1^{**} which would satisfy conditions

$$\Phi_1^{**} \leqslant \Phi_1 \leqslant \Phi_1^*,$$
$$\lim |\Phi_1^* - \Phi_1^{**}| = 0,$$
$$\overline{\mathrm{Pe}} \to +\infty.$$

Let us assume that in solution (9) $\delta = 0$. This results in a lowering of the temperature at all points of the rod, except at $x = 0$ and $x = +\infty$, where the temperature remains unchanged. For small x the gradient of the new field

$$U(x,\ t) = U_0 \left[\exp\left(-\frac{v_1 x}{a}\right) - \exp\left(-\frac{v_1 x}{2a}\right) \operatorname{erf}\left(\frac{x}{2\sqrt{at}}\right)\right] \tag{11}$$

is greater than in (9). Such a temperature field determines the heat flux Φ_1^* which is an estimate of the upper limit of the actual flux Φ_1.

The lower limit is derived from the temperature field

$$U(x,\ t) = U_0 \exp\left(-\frac{v_1 x}{a}\right), \tag{12}$$

which is the first term of (9). For small x the gradient of field (12) is smaller than that of (9). Hence the heat flux Φ_1^{**} calculated by formula (10) with (12) taken into consideration defines the lower limit of possible values of Φ_1.

The expressions for heat fluxes Φ_1^* and Φ_1^{**} in dimensionless parameters are of the form

$$Q_1^* = 1 - \exp(-\overline{\mathrm{Pe}}) - \frac{1}{2} \int_0^{\overline{\mathrm{Pe}}} \exp\left(-\frac{\mathrm{Pe}}{2}\right) \operatorname{erf}(Z_1)\, d\,\mathrm{Pe} + \frac{1}{\sqrt{\pi}} \int_0^{\overline{\mathrm{Pe}}} \frac{\exp\left(-\frac{\mathrm{Pe}}{2} - Z_1^2\right)}{\sqrt{\overline{\mathrm{Pe}} - \mathrm{Pe}}}\, d\,\mathrm{Pe}, \tag{13}$$

$$Q_1^{**} = 1 - \exp(-\overline{\mathrm{Pe}}); \tag{14}$$

here $Q_1^* = \Phi_1^*/U_0\lambda bc\tan\beta$ and $Q_1^{**} = \Phi_1^{**}/U_0\lambda bc\tan\beta$ are numbers proportional to heat fluxes Φ_1^* and Φ_1^{**}, respectively; $\mathrm{Pe} = v_1 x/a$ is the Peclet number; $\overline{\mathrm{Pe}} = v_1 h/a\cos\beta$ is the maximum value of the Peclet number in a given process; $Z_1 = \mathrm{Pe}/2\sqrt{\overline{\mathrm{Pe}} - \mathrm{Pe}}$ is the value of complex Z at the shear line.

With increasing Peclet number, functions Q_1^* and Q_1^{**} tend asymptotically to unity (Fig. 2). When $\overline{Pe} > 5$, it may be assumed that $Q_1^* \approx Q_1^{**} \approx Q_1 = 1$. In such cases the heat flux reaching the workpiece from the chip-formation zone is calculated by formula

$$\Phi_1 = U_0 \lambda b \operatorname{ctg} \beta. \tag{15}$$

The depth of cut and the cutting speed do not appear in (15). At $\overline{Pe} > 5$ these parameters affect the heat flux only if they vary the angle β of inclination of the strain zone.

Equation (15) shows that the heat flux is directly proportional to the temperature of the nominal slip plane; and the variation of the heat flux affects, in turn, the temperature of U_0. To elucidate this effect we calculate the mean temperature in the strain zone from the heat balance of the latter:

$$\begin{gathered} \Phi = \Phi_1 + \Phi_2, \\ A_W hbv = U_0 \lambda b \operatorname{ctg} \beta + U_0 hbvc_W, \\ U_0 = \frac{A_W}{c_W + (hv)^{-1} \lambda \operatorname{ctg} \beta}. \end{gathered} \tag{16}$$

Let the angle β of inclination of the strain zone tend to zero with constant parameters v, λ, h, and b. In accordance with (15) the heat flux will then tend to increase infinitely. This becomes evident, if one finds the limit of expression (16) for $\beta \to 0$, and then elucidates the behavior of the heat flux. Using the expression known in the theory of cutting, we obtain

$$\lim_{\beta \to 0} U_0 = \lim_{\beta \to 0} \frac{\sigma [\operatorname{ctg} \beta + \operatorname{tg} (\beta - \gamma)]}{c_W + (hv)^{-1} \lambda \operatorname{ctg} \beta} = \frac{\sigma hv}{\lambda}. \tag{17}$$

Substituting (17) into (15), we conclude that the heat flux infinitely increases.

Let now the angle of inclination of the strain zone increase. When $\beta \to \pi/2$, the heat flux reaching the workpiece tends to zero. At high cutting speeds and increased angle β the second term in the denominator in formula (16) may be neglected, and the temperature calculated by the simpler formula

$$U_0 = A_W c_W^{-1}.$$

The validity of formula (15) was checked by special tests. Cylindrical test pieces, made from grade 35 steel, were turned in reverse cuts and their heat content was measured. This made possible the calculation of the total heat flux to the workpiece. In these tests the angle β of the strain zone inclination was held approximately constant at 25°. Machining was by a square-nosed cutter 2.1 mm wide with a T15K6 hard alloy tip ($\gamma = 10°$, $\alpha = 10°$). The depth of cut was 0.3 mm and the feed 2.08 mm/rev.

Under these machining conditions the heat flux to the workpiece may be considered as the sum

$$\Phi_\Sigma = \Phi_1 + \Phi_3 \tag{18}$$

of two fluxes: the flux from the strain zone and the one due to friction at the relief flank of the cutter. The latter may be expressed by

$$\Phi_3 = Fv. \tag{19}$$

The first term of (18) will be constant, if in a given series of tests $\overline{Pe} > 5$, and parameters U_0, b, and β are constant. In this case the curve of Φ_Σ is obtained from (19) by adding Φ_1 = const. Taking into account that $\Phi_3 = 0$ when v = 0, it is possible to determine Φ_1 by extrapolating the experimentally obtained dependence (18) to zero cutting speed. The segment cut off on the Φ-axis gives the value of the heat flux from the strain zone to the workpiece.

Experimental results are shown in Fig. 3. The heat flux from the strain zone to the workpiece calculated by formula (15) for the same conditions of cutting is 72 W, which is in good correlation with experimental data.

NOTATION

v	is the velocity of the medium;
v_1, v_2	are components of the medium velocity;
h, b	are the depth and width of cut, respectively;
y	is the coordinate defining the rod position;
t	is the time of heating of the rod;

x	is the coordinate of the point of temperature calculation;
U_0	is the mean temperature of the slip surface (idealized stress zone);
a, λ	are the coefficients of thermal diffusivity and conductivity, respectively;
c_W	is the volumetric specific heat of the machined material;
A_W	is the specific strain work;
σ	is the tangential stress in the nominal slip plane;
Φ	is the thermal power of the strain zone;
Φ_1	is the heat flux to the workpiece from the strain zone;
Φ_2	is the heat flux to the chip;
Φ_3	is the heat flux to the workpiece generated by the cutter clearance flank against the workpiece;
Φ_Σ	is the total flux reaching the workpiece;
F	is the force of friction between the cutter clearance flank and the workpiece;
β	is the angle between the nominal slip plane and the shear plane.

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